

# Seismic Response History Procedure

## A Program for Nonlinear Structural Analysis

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## 1. Introduction

The Seismic Response History Procedure (SRHP) is a determined nonlinear structural analysis software, based on the most current IBC/CBC, ASCE, ACI and AASHTO, without probability and/or fuzzy math. The SRHP is also an open system, which the element matrix, design criteria, and even nonlinear method, are all changeable. From the manual example, user can find a 5 story building, under El Centro 1940 earthquake, history procedures of story drift, equivalent base shear and later forces, and their maximum value with its happened time.

## 2. Equation of Motion

The seismic analysis/design is based on the following equation of motion.

$$[m]\{\ddot{u}\} + [c]\{\dot{u}\} + [k]\{u\} = \{f_g\} - [m]\{\ddot{u}_g\} \quad (\text{Eq. 2.1})$$

Where:

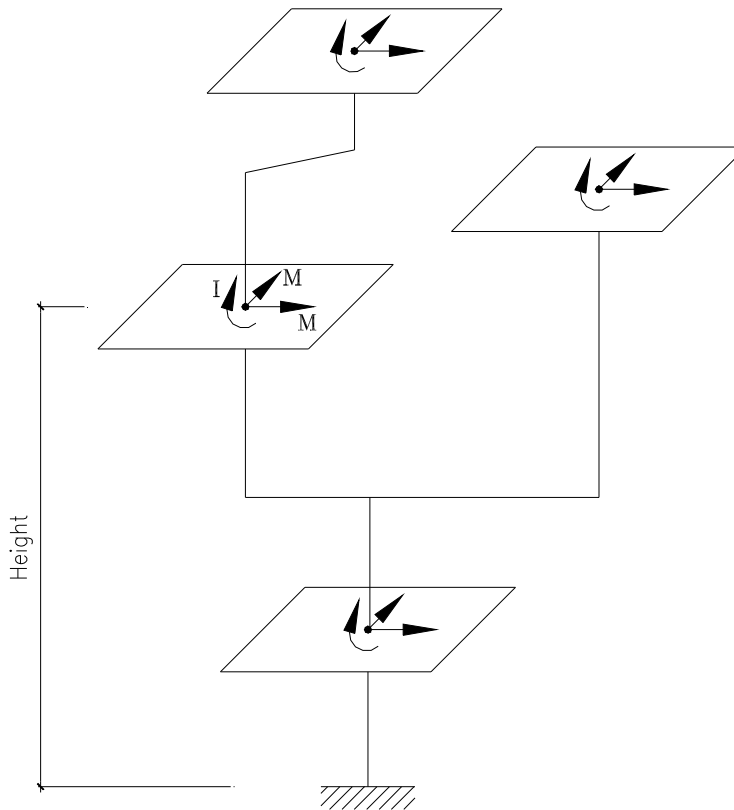


Fig. 2.1 Global Structural DOFs in Equation of Motion

$[m]$  = Diagonal mass matrix based on each floor diaphragm center points (selected global DOFs), including horizontal X, Y directions, and  $\Phi$  moment of inertia. The each floor diaphragm center points may not be at a same vertical location to keep mass matrix diagonal.

The moment of inertia is a rigid diaphragm concept. Semirigid modeling assumption (ASCE 7-10 12.3.1) forces the  $[m]$  to non-diagonal matrix, which results from Complex Eigenvector modes. This software user can cut a diaphragm to two and more, but each smaller one has to be rigid.

$\{u\}$  = Displacement vector at each floor diaphragm center points, including horizontal X, Y directions, and  $\Phi$  rotation. Typical for velocity  $\{\dot{u}\}$  and acceleration  $\{\ddot{u}\}$ .

$[k]$  = Lateral stiffness full matrix based on each floor diaphragm center points, which concentrated from each vertical 2D frames.

$\{\ddot{u}_g\}$  = Ground acceleration, including horizontal X, Y directions, and  $\Phi$  rotation, without SRSS probability issue. User can rotate the structural locations to get maximum responses.

To get the ground motions in a maximum direction (ASCE 7-10 16.1.3.2) is based on Single Degree of Freedom, because any actual structural stiffness,  $[k]$ , is full matrix, which means that the two horizontal X and Y responses coupled together. One of DOFs at one direction reached maximum response does not mean other all DOFs maximum responses, even minimum at the same direction.

$[c]$  = Damping matrix as follows.

$$[c] = \frac{4\pi(\xi)}{T_1 + T_2} [m] + \frac{(\xi)T_1 T_2}{\pi(T_1 + T_2)} [k] \quad (\text{Eq. 2.2})$$

The reasons that Eq. 2.2 has to be applied are

1. Only damping ratio of  $\zeta$  has been called out, 5%, on ASCE 7-10, 16.1.3 & 21.1.3. There are no other adapted law document for damping input. The (Eq. 2.2) has reached the code requirement.
2. The (Eq. 2.2) is an applicable math method to solve the equation of motion (Eq. 2.1), because structural period  $T_1$  &  $T_2$  are not constants in nonlinear structural analysis. The  $T_1$  &  $T_2$  are changed in each time steps after plastic hinges formed.

$\{f_g\}$  = Must be Zero vector. Otherwise, the equation (Eq. 2.1) cannot be solved as classical damped system. The static gravity loads are not vectors changed on time steps in Equation of Motion.

### 3. Lateral Resisting Frame

The lateral resisting frames are 2D vertical substructures.

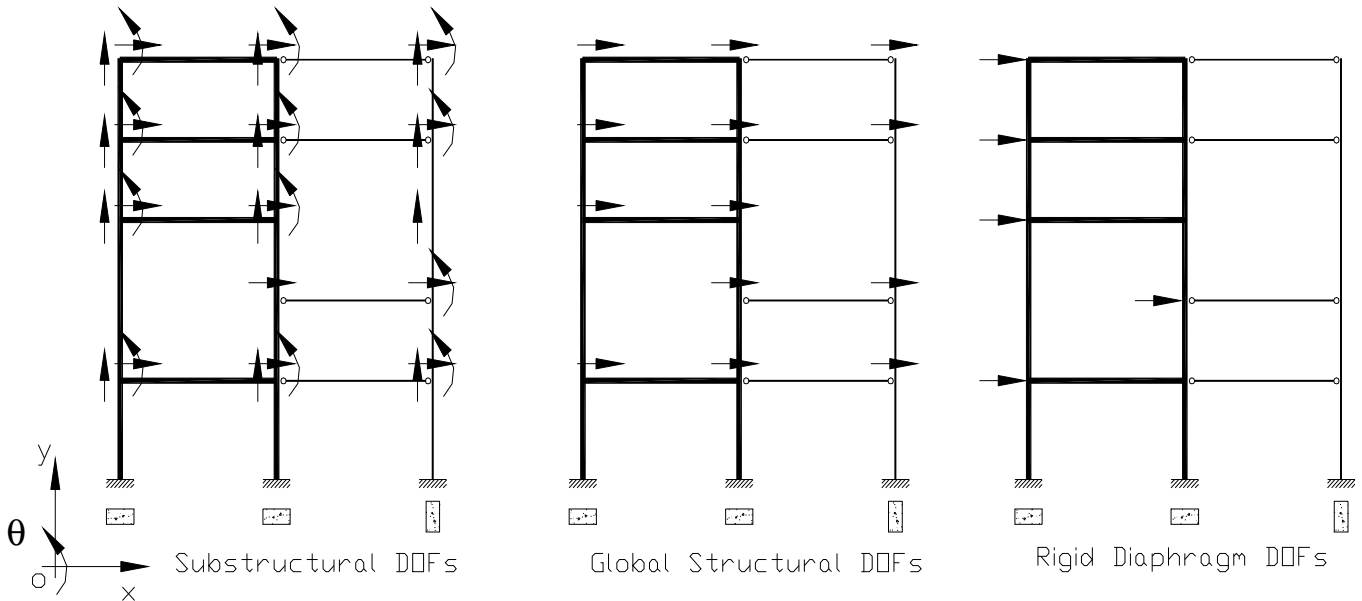


Fig. 3.1 Substructural DOFs for a 2D Lateral Resisting Frame

The reasons to use 2D lateral resisting frame, not 3D, as substructure are

1. For steel design, the Special Moment Frame (SMF) is based on 2D requirements of AISC 341-05 and AISC 358-05, and orthogonal moment frames sharing common column are not permitted by 2010 CBC 2205A.5.
2. For concrete design, the biaxial bending cannot be separated. If orthogonal moments exist con-currently, the ACI 318-08 Chapter 21 cannot be applied.

### 4. Finite Element

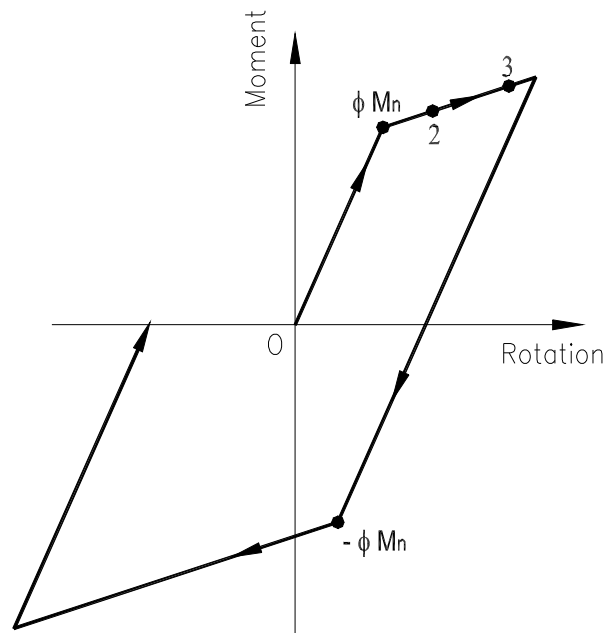
Nonlinear Beam-Column element:

1. Concrete beam/column/brace
2. Steel beam/column/brace
3. Super composite column

Linear Wall/Diaphragm element.

The following finite elements are all changeable.

Fig. 4.1 Elastoplastic Relation ==>



# TYPICAL BEAM- COLUMN ELEMENT

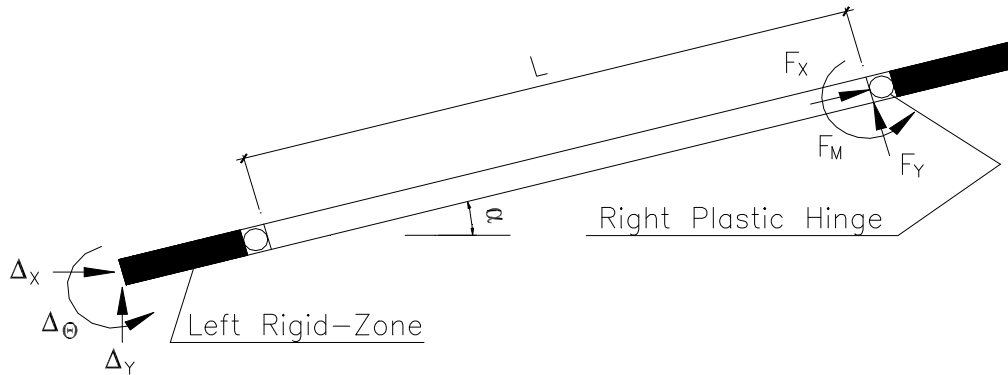
UNIT = 1 ( U.S. Customary System)  
 SECTION = RC-1

JOINTS = i to j  
 X = 0 ft to 0 ft  
 Y = 16 ft to 26 ft

$\alpha$  = 90 °

E = 4030.50865 ksi  
 G = 1550.19564 ksi  
 A = 1080 in<sup>2</sup>  
 I = 116640 in<sup>2</sup>\*2  
 k = 1.2 , (1.2 for rectangular section, 10/9 for circular section.)  
 $\beta$  =  $12 E I k / (G A L^2) = 2.82371095$

L<sub>Left Rigid</sub> = 1.369 ft  
 L<sub>Left Hinge</sub> = 1.369 ft <== 15% E , Plastic Hinge ? ==> 0.15  
 L = 3.153 ft , (100% for moment connection, 0% pinned, 0% to 100% for plastic hinge)  
 L<sub>Right Hinge</sub> = 2.739 ft <== 15% E , Plastic Hinge ? ==> 0.15  
 L<sub>Right Rigid</sub> = 1.369 ft



F = [Element Coordinate] = (kips, in)	-140.3248	Axial, Left	Δ = [Frame Coordinate] = (kips, in)	0.0006	X
	16.0284	Shear		-0.0780	Y
	752.9447	Moment		0.0002	θ
	140.3248	Axial, Right		0.0013	X
	-16.0284	Shear		-0.0909	Y
	-146.4023	Moment		0.0000	θ

[K] = [T]<sup>T</sup> [k] [T] =  
(kips, in)

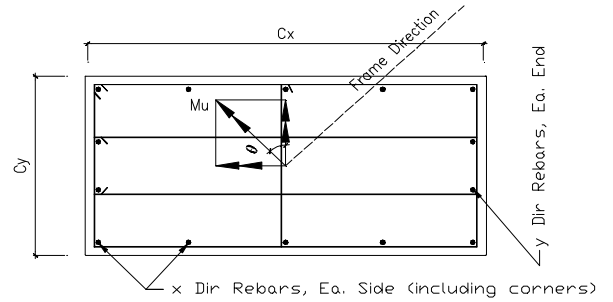
	i	i	i	j	j	j
i	1115.763618	5.99382E-13	-71868.25492	-1115.763618	-5.99382E-13	-62023.37924
i	5.99382E-13	10900.39978	4.40246E-12	-5.99382E-13	-10900.39978	3.79939E-12
i	-71868.25492	4.40246E-12	5806401.165	71868.25492	-4.40246E-12	2817789.426
j	-1115.763618	-5.99382E-13	71868.25492	1115.763618	5.99382E-13	62023.37924
j	-5.99382E-13	-10900.39978	-4.40246E-12	5.99382E-13	10900.39978	-3.79939E-12
j	-62023.37924	3.79939E-12	2817789.426	62023.37924	-3.79939E-12	4625016.083

# Concrete Section Design Based on ACI 318-08

Section RC-1  
No. 1

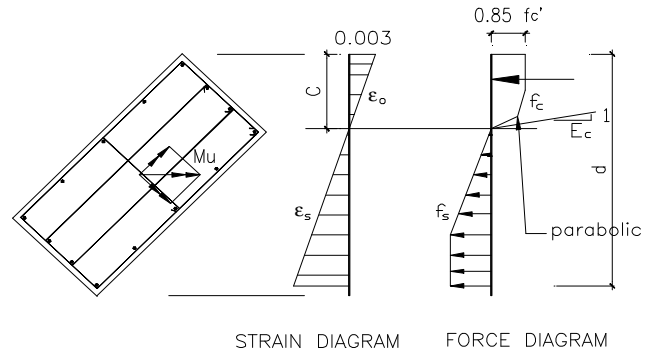
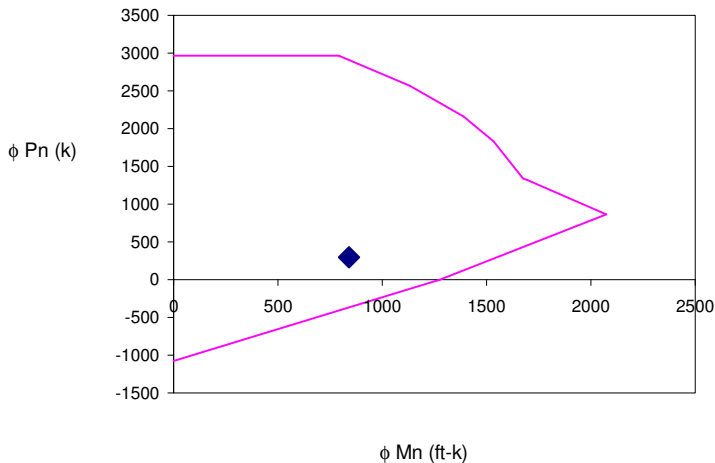
## INPUT DATA & DESIGN SUMMARY

CONCRETE STRENGTH  $f'_c = 5$  ksi  
 REBAR YIELD STRESS  $f_y = 60$  ksi  
 SECTION SIZE  $C_x = 36$  in  
 $C_y = 30$  in  
 FACTORED AXIAL LOAD  $P_u = 300$  k  
 FACTORED MAGNIFIED MOMENT  $M_u = 840.9$  ft-k  
 VERT. REINFORCEMENT 7 # 9 at x dir. (Total 20 # 9)  
 3 # 9 at y dir.  
 LATERAL FRAME DIRECTION  $\theta = 0$  deg



## Linear Stage

## ANALYSIS



	$\phi P_n$ (k)	$\phi M_n$ (ft-k)
AT AXIAL LOAD ONLY	2967	0
AT MAXIMUM LOAD	2967	790
AT 0 % TENSION	2570	1131
AT 25 % TENSION	2159	1390
AT 50 % TENSION	1830	1532
AT $\epsilon_t = 0.002$	1342	1674
AT BALANCED CONDITION	1323	1695
AT $\epsilon_t = 0.005$	866	2074
AT FLEXURE ONLY	0	1276
AT PURE TENSION	-1080	0

$$\epsilon_o = \frac{2(0.85 f'_c)}{E_c}, \quad E_c = 57\sqrt{f'_c}, \quad E_s = 29000 \text{ ksi}$$

$$f_c = \begin{cases} 0.85 f'_c \left[ 2 \left( \frac{\epsilon_c}{\epsilon_o} \right) - \left( \frac{\epsilon_c}{\epsilon_o} \right)^2 \right], & \text{for } 0 < \epsilon_c < \epsilon_o \\ 0.85 f'_c, & \text{for } \epsilon_c \geq \epsilon_o \end{cases}$$

$$f_s = \begin{cases} \epsilon_s E_s, & \text{for } \epsilon_s \leq \epsilon_t \\ f_y, & \text{for } \epsilon_s > \epsilon_t \end{cases}$$

**WF, Tube, or Pipe Design Based on AISC 360-05**

Section **ST-1**  
 No. **1**

**INPUT DATA & DESIGN SUMMARY**

MEMBER SHAPE (WF, Tube, or Pipe) & SIZE

**W24X250** <== **W Shape**

STEEL YIELD STRESS  $F_y = 50$  ksi

AXIAL COMPRESSION FORCE  $P_r = -531.06$  kips, ASD

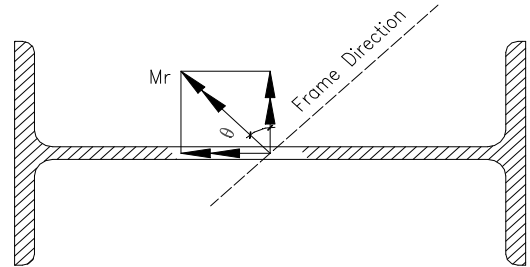
LATERAL BENDING MOMENT  $M_r = 840.9$  ft-kips, ASD

LATERAL FRAME DIRECTION  $\theta = 0$  deg

STRONG AXIS EFFECTIVE LENGTH  $kL_x = 16$  ft

WEAK AXIS EFFECTIVE LENGTH  $kL_y = 16$  ft

STRONG AXIS BENDING UNBRACED LENGTH  $L_b = 16$  ft, (AISC 360-05 F2.2.c)



**Linear Stage**

**ANALYSIS**

CHECK COMBINED COMPRESSION AND BENDING CAPACITY (AISC 360-05, H1)

$$\left\{ \begin{array}{l} \frac{P_r}{P_c} + \frac{8}{9} \left( \frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}} \right), \text{ for } \frac{P_r}{P_c} \geq 0.2 \\ \frac{P_r}{2P_c} + \left( \frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}} \right), \text{ for } \frac{P_r}{P_c} < 0.2 \end{array} \right. = 0.23 < 1.0 \quad \text{[Satisfactory]}$$

Where  $M_{rx} = 840.90$  ft-kips, ASD

$M_{ry} = 0.00$  ft-kips, ASD

$P_c = P_n / \Omega_c = 2962 / 1.67 = 1773.86$  kips, (AISC 360-05 Chapter E)

>  $P_r$  **[Satisfactory]**

$M_{cx} = M_n / \Omega_b = 3721.84 / 1.67 = 2228.65$  ft-kips, (AISC 360-05 Chapter F)

>  $M_{rx}$  **[Satisfactory]**

$M_{cy} = M_n / \Omega_b = 891.67 / 1.67 = 533.93$  ft-kips, (AISC 360-05 Chapter F)

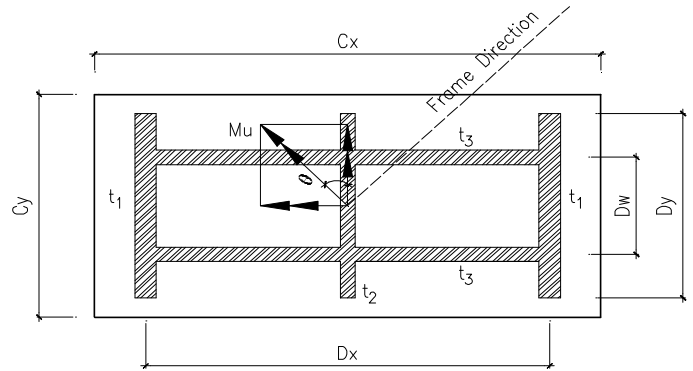
>  $M_{ry}$  **[Satisfactory]**

# Super Composite Column Design Based on AISC 360-05 & ACI 318-08

Section SC-1  
No. 1

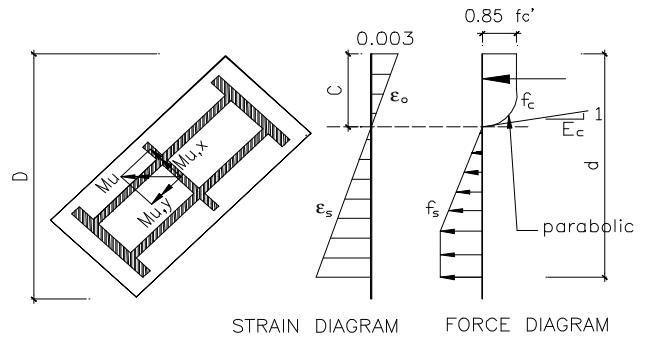
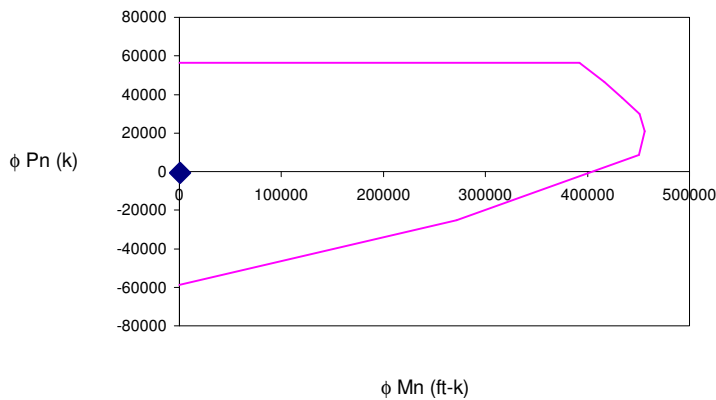
## INPUT DATA & DESIGN SUMMARY

CONCRETE STRENGTH	$f'_c =$	5	ksi
STEEL YIELD STRESS	$f_y =$	50	ksi
COLUMN EFFECTIVE LENGTH	$KL =$	240	ft
CONCRETE SECTION SIZE	$C_x =$	240	in
	$C_y =$	125	in
STEEL SECTION SIZE	$D_x =$	192	in
	$D_y =$	100	in
	$t_1 =$	2	in
	$t_2 =$	1.5	in
	$t_3 =$	2	in
	$D_w =$	75	in
FACTORED AXIAL LOAD	$P_u =$	-531.1	k
FACTORED MOMENT	$M_u =$	840.9	ft-k
LATERAL FRAME DIRECTION	$\theta =$	0	deg



Linear Stage

## ANALYSIS



Capacity Drawings	$\phi$	$\phi P_n$ (k)	$\phi M_n$ (ft-k)
AT AXIAL LOAD ONLY	0.75	56495	0
AT MAXIMUM LOAD	0.75	56495	392406
AT AXIAL LOAD 46347 k	0.75	46347	416294
AT AXIAL LOAD 38312 k	0.771	38312	434233
AT AXIAL LOAD 29956 k	0.811	29956	450761
AT AXIAL LOAD 20768 k	0.85	20768	455953
AT STEEL STRAIN 0.005	0.9	8533	450477
AT AXIAL LOAD -25073 k	0.9	-25073	271881
AT PURE TENSION	0.9	-58680	0

$$\epsilon_o = \frac{2(0.85 f'_c)}{E_c}, \quad E_c = 57\sqrt{f'_c}, \quad E_s = 29000 \text{ ksi}$$

$$f_c = \begin{cases} 0.85 f'_c \left[ 2 \left( \frac{\epsilon_c}{\epsilon_o} \right) - \left( \frac{\epsilon_c}{\epsilon_o} \right)^2 \right], & \text{for } 0 < \epsilon_c < \epsilon_o \\ 0.85 f'_c, & \text{for } \epsilon_c \geq \epsilon_o \end{cases}$$

$$f_s = \begin{cases} \epsilon_s E_s, & \text{for } \epsilon_s \leq \epsilon_t \\ f_y, & \text{for } \epsilon_s > \epsilon_t \end{cases}$$



## TYPICAL SHEAR WALL / DIAPHRAGM ELEMENT

UNIT = 1 ( U.S. Customary System)

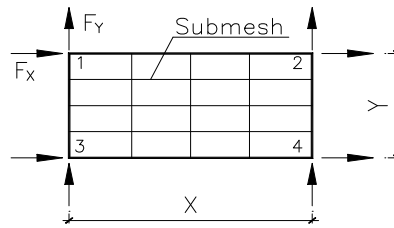
JOINTS = 1 2 3 4

X = 288 in  
Y = 120 in

t = 8 in, (thickness)  
E = 3320.6 ksi

( $w_c^{1.5} 33 f_c^{0.5}$  for concrete, 29000 ksi for steel)

$\nu$  = 0.25, (Poisson's ratio)



	1 x	1 y	2 x	2 y	3 x	3 y	4 x	4 y	
$[K] = [k_{11}] - [k_{12}] [k_{22}]^{-1} [k_{21}] =$ (kips, in)	6174.922507	-1211.24393	-956.3157582	269.7714154	-1969.676208	-963.1755491	-3248.930541	1904.648063	1 x
	-1211.24393	9967.162057	-74.77244726	-48.59230415	-618.6316864	-9431.3219	1904.648063	-487.2478529	1 y
	-956.3157582	-74.77244726	6414.402796	2892.834619	-3488.41083	-2199.430485	-1969.676208	-618.6316864	2 x
	269.7714154	-48.59230415	2892.834619	10749.07441	-2199.430485	-1269.16021	-963.1755491	-9431.3219	2 y
	-1969.676208	-618.6316864	-3488.41083	-2199.430485	6414.402796	2892.834619	-956.3157582	-74.77244726	3 x
	-963.1755491	-9431.3219	-2199.430485	-1269.16021	2892.834619	10749.07441	269.7714154	-48.59230415	3 y
	-3248.930541	1904.648063	-1969.676208	-963.1755491	-956.3157582	269.7714154	6174.922507	-1211.24393	4 x
	1904.648063	-487.2478529	-618.6316864	-9431.3219	-74.77244726	-48.59230415	-1211.24393	9967.162057	4 y

$[F] =$ (kips)	-35.1828	1 x
	-59.6315	1 y
	-30.3125	2 x
	-19.1750	2 y
	41.0183	3 x
	86.9212	3 y
	24.4770	4 x
	-8.1147	4 y

$[\Delta] =$ (in)	0.0020	1 x
	0.0030	1 y
	0.0040	2 x
	0.0100	2 y
	0.0100	3 x
	0.0100	3 y
	0.0100	4 x
	0.0100	4 y

## 5. Input Data

The input data include structural information and ground acceleration, as shown on the following example.

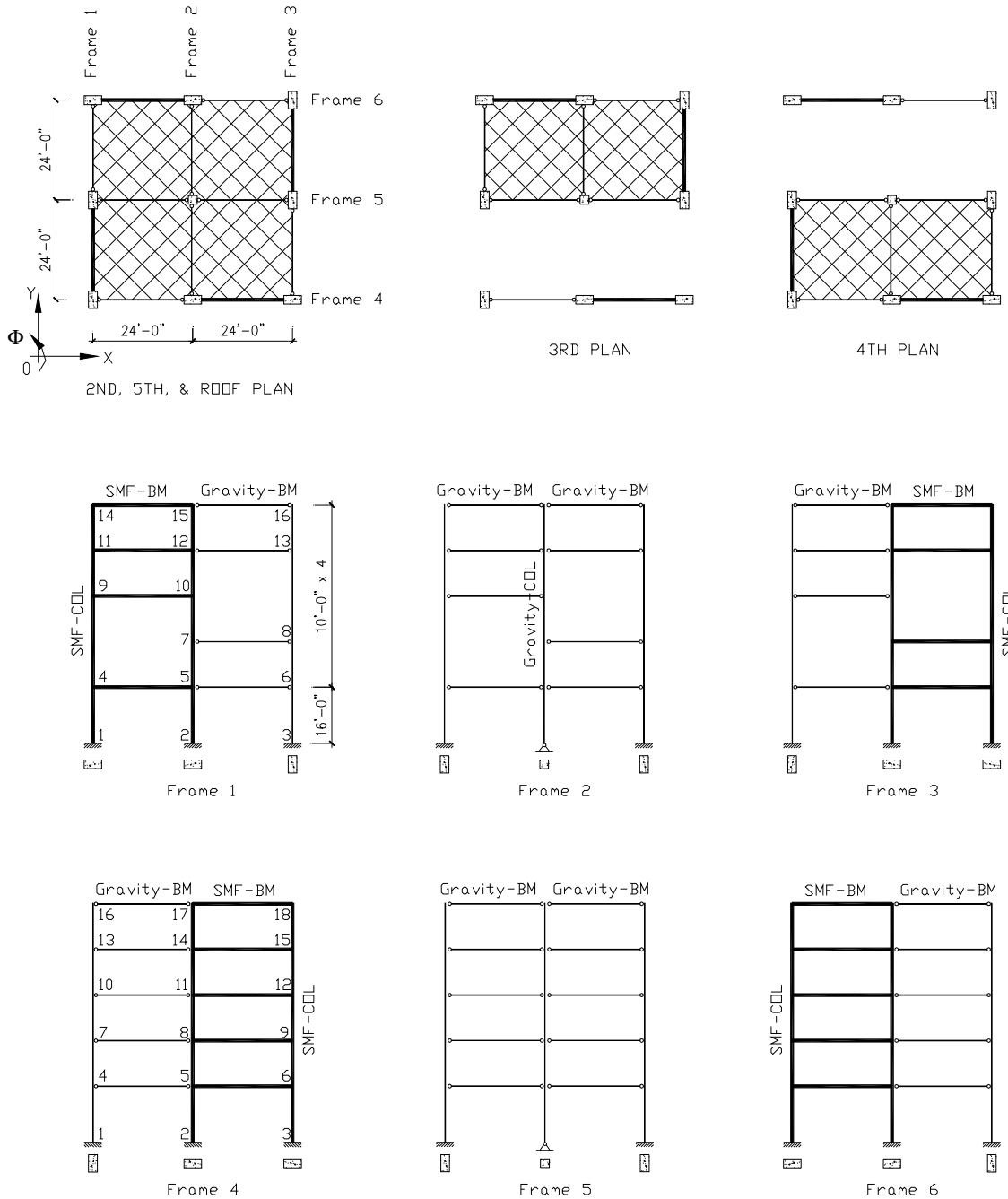


Fig. 5.1 Floor Plan & Frame Elevation

SMF-COL = 30" x 36" ,20 # 9 (7 # 9 at Bending Side), 4 Legs - # 5 @ 4" O.C. (ACI 318 21.6)  
 SMF-BM = 24" x 36" ,9 # 9 Top 6 # 9 Bot., 5 Legs - # 5 @ 8" O.C. (ACI 318 21.6)









